International Journal of Research in Advent Technology, Vol.6, No.7, July 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces

V.K.Shanthi[1], S.Chandrasekar[2], K.Safina Begam[3]
[1] Associate Professor, Department of Mathematics,
Sri Meenakshi Government Arts College for Women,
Madurai(DT),Tamil Nadu, India.
[2] Assistant Professor, PG and Research Department of Mathematics,
Arignar Anna Government Arts College,
Namakkal(DT),Tamil Nadu, India.
[3]Part time Research scholar,
Madurai Kamaraj University, Tamilnadu, India.
Email: vksanthimadurai @yahoo.co. in[1] , chandrumat@gmail.com[2] safinabegam2009@gmail.com[3]

Abstract- F.Smarandache introduced and developed the concept of Neutrosophic set from the fuzzy sets and intuitionistic fuzzy sets . A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. Aim of this paper is we introduced and studied about Neutrosophic generalized semi closed sets in Neutrosophic topological spaces and its properties are discussed details

Index Terms- *Neutrosophic semi closed sets, Neutrosophic semi open sets, Neutrosophic generalized semi closed sets, Neutrosophic generalized semi open sets*

1. INTRODUCTION

Topology is a classical subjects, as a generalization topological spaces many type of topological spaces introduced over the year. C.L. Chang[2] was introduced and developed fuzzy topological space by using L.A. Zadeh's[14] fuzzy sets. Coker[5] introduced the notion of Intuitionistic fuzzy topological spaces by using Atanassov's[1] intuitionistic fuzzy set

Neutrality the degree of indeterminacy, as an independent concept, was introduced by Smarandache [8] in 1998. He also defined the Neutrosophic set on three component Neutrosophic topological spaces (t, f ,i) =(Truth, Falsehood, Indeterminacy),The Neutrosophic crisp set concept was converted to Neutrosophic topological spaces by Salama [12]et al.

R.Dhavaseelan[4],SaiedJafari are introduced Neutrosophic generalized closed sets.K. Bageerathi [10]et al introduced and studied about Neutrosophic semi closed sets in Neutrosophic topological spaces

In this paper we introduced and studied about Neutrosophic generalized semi closed sets in Neutrosophic topological spaces and its properties are discussed details

2. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operation.

Definition 2.1 [9]

Let X be a non-empty fixed set. A Neutrosophic set A is an object having the form

$$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$$

where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent Neutrosophic topological spaces the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.2 [9]

A Neutrosophic set

$A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$

can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A \rangle$ in]-0,1+[on X.

Remark 2.3[9]

we shall use the symbol

$A =\langle x, \mu_A, \sigma_A, \gamma_A \rangle$ for the Neutrosophic set

$A =\{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle :x \in X\}$.

Example 2.4 [9]

Every Intuitionistic fuzzy set A is a non-empty set in X is obviously on Neutrosophic set having the form

$A = \{ \langle x, \mu_A(x), 1-(\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle : x \in X \}$.

Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic set 0_N and 1_N in X as follows:

0_N may be defined as :

(0_1) $0_N =\{\langle x, 0, 0, 1\rangle : x \in X\}$

(0_2) $0_N =\{\langle x, 0, 1, 1\rangle : x \in X\}$

(0_3) $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$

(0_4) $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$

1_N may be defined as :

(1_1) $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$

(1_2) $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$

(1_3) $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$

(1_4) $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$

Definition 2.5 [9]
Let A = ⟨ μ_A, σ_A, γ_A ⟩ be a Neutrosophic set on X, then the complement of the set A
[C(A)for short] defined as
C(A)={⟨ x, $\gamma_A(x)$, 1 - $\sigma_A(x)$, $\mu_A(x)$⟩: x∈X}

Definition 2.6 [9]
Let x be a non-empty set, and Neutrosophic sets A and B in the form
A = {⟨ x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$⟩: x∈X} and
B = {⟨ x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$⟩: x∈X}.
Then we consider definition for subsets (A⊆B).
A⊆B defined as :
A⊆ B ⇔ $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and
$\gamma_A(x) \geq \gamma_B(x)$ for all x∈X

Proposition 2.7 [9]
For any Neutrosophic set A, then the following condition are holds :
(i) 0_N ⊆A , $0_N \subseteq 0_N$
(ii) A⊆1_N , $1_N \subseteq 1_N$

Definition 2.8 [9]
Let X be a non-empty set, and
A=⟨ x, $\mu_A(x)$,$\sigma_A(x)$, $\gamma_A(x)$⟩ ,
B =⟨x, $\mu_B(x)$, $\sigma_B(x)$, $\gamma_B(x)$⟩ are Neutrosophic sets.
Then
(i) A∩B defined as :
A∩B =⟨ x, $\mu_A(x)\wedge\mu_B(x)$, $\sigma_A(x)\wedge\sigma_B(x)$,$\gamma_A(x)\vee\gamma_B(x)$⟩
(ii) A∪B defined as :
A∪B =⟨x, $\mu_A(x)\vee\mu_B(x)$, $\sigma_A(x)\vee\sigma_B(x)$, $\gamma_A(x)\wedge\gamma_B(x)$⟩

Definition 2.9 [9]
We can easily generalize the operation of intersection and union in Definition 2.8 to arbitrary family of Neutrosophic sets as follows :
Let { A_j : j ∈ J } be a arbitrary family of Neutrosophic sets in X, then
(i) ∩Aj defined as :
∩A_j =⟨ x, $\wedge_{j\in J}\mu_{Aj}(x)$,$\wedge_{j\in J}\sigma_{Aj}(x)$,$\vee_{j\in J}\gamma_{Aj}(x)$ ⟩
(ii) ∪Aj defined as : ∪Aj=⟨ x, ⋁, ⋁,⋀ ⟩

Proposition 2.10 [9]
For all A and B are two Neutrosophic sets then the following condition are true :
(1) C(A∩B)=C(A)∪C(B)
(2) C(A∪B)=C(A)∩C(B).

Definition 2.11 [11,12]
A Neutrosophic topology is a non -empty set X is a family τ_N of Neutrosophic subsets in X satisfying the following axioms :
(i) 0_N,1_N ∈ τ_N ,
(ii) $G_1 \cap G_2 \in \tau_N$ for any G_1, $G_2 \in \tau_N$,
(iii) ∪Gi ∈ τ_N for every {Gi:i ∈J}⊆τ_N
the pair(X, τ_N) is called a Neutrosophic topological space.
The element Neutrosophic topological spaces of τ_N are called Neutrosophic open sets.
A Neutrosophic set F is closed if and only if C(F) is Neutrosophic open.

Example 2.14 [11,12]
Let X={x} and
A_1 = {⟨ x, 0.6, 0.6, 0.5 ⟩ : x ∈X}
A_2 = {⟨ x, 0.5, 0.7, 0.9 ⟩ : x ∈X}
A_3 = {⟨ x, 0.6, 0.7, 0.5 ⟩ : x ∈X}
A_4 = {⟨ x, 0.5, 0.6, 0.9 ⟩ : x ∈X}
Then the family τ_N={0_N, A_1, A_2, A_3, A_4, 1_N}is called a Neutrosophic topological space on X.

Definition 2.15 [11,12] Let(X, τ_N) be Neutrosophic topological spaces and
A ={⟨ x, $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ ⟩:x∈X }
be a Neutrosophic set in X.
Then the Neutrosophic closure and Neutrosophic interior of A are defined by
Neu-Cl(A) =∩{ K : K is a Neutrosophic closed set in X and A ⊆ K}
Neu-Int(A) = ∪{G : G is a Neutrosophic open set in X and G ⊆ A}.

Definition 2.16 [11,12]
(i) A is Neutrosophic open set if and only if A=Neu-Int(A) .
(ii) A is Neutrosophic closed set if and only if A=Neu-Cl(A).

Proposition 2.17 [11,12]
For any Neutrosophic set A in(X, τ_N) we have
(i) Neu-Cl(C(A))=C(Neu-Int(A)),
(ii) Neu-Int(C(A))=C(Neu-Cl(A)).

Proposition 2.18 *[11,12]*
Let(X, τ_N) be a Neutrosophic topological spaces and A, B be two Neutrosophic sets in X. Then the following properties are holds :
(i) Neu-Int(A)⊆A,
(ii) A ⊆ Neu-Cl(A),
(iii) A⊆B ⇒ Neu-Int(A)⊆Neu-Int(B),
(iv) A⊆B⇒Neu-Cl(A)⊆Neu-Cl(B),
(v) Neu-Int(Neu-Int(A))=Neu-Int(A),
(vi) Neu-Cl(Neu-Cl(A))=Neu-Cl(A),
(vii) Neu-Int(A∩B))=Neu-Int(A)∩Neu-Int(B),
(viii) Neu-Cl(A∪B)=Neu-Cl(A)∪Neu-Cl(B),
(ix) Neu-Int(0_N)=0_N ,
(x) Neu-Int(1_N)=1_N ,
(xi) Neu-Cl(0_N)=0_N ,
(xii) Neu-Cl(1_N)=1_N ,
(xiii) A⊆B⇒C(B)⊆C(A),
(xiv) Neu-Cl(A∩B)⊆Neu-Cl(A)∩Neu-Cl(B),
(xv) Neu-Int(A∪B)⊇Neu-Int(A)∪Neu-Int(B).

Definition:2.19 [10]
A subset A of a Neutrosophic space(X, τ_N) is called Neutrosophic semi-open
if A⊆ Neu-Cl(Neu-int(A)).
The complement of Neutrosophic semi-open set is called Neutrosophic semi-closed.

Definition 2.20 [4]
Let A be a subset of a Neutrosophic space (X, τ_N) is called generalized Neutrosophic closed(Neu g-closed) if Neu-clA⊆U,whenever A⊆U and U is Neu-open.
The complement of a Neu g- closed set is called the Neu g-open set.

3. NEUTROSOPHIC GENERALIZED SEMI CLOSED SET IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the Neutrosophic generalized semi closed set in Neutrosophic topological spaces

Definition 3.1

Let A be a subset of a Neutrosophic space(X, τ_N) is called Neutrosophic generalized semi closed(Neu-GS-closed) if Neutrosophic semi-clA⊆U, whenever A⊆U and U is Neutrosophic open.

The complement of a Neu-GS-closed set is called the Neu-GS-open set.

Example 3.2

Let X = { *a* , *b* } and

A_1 = ⟨(0.4, 0.6, 0.5),(0.7, 0.3, 0.6)⟩

A_2 = ⟨(0.3, 0.7, 0.8),(0.6, 0.4, 0.2)⟩

A_3 = ⟨(0.4, 0.7, 0.5),(0.7, 0.4, 0.2)⟩

A_4 = ⟨(0.3, 0.6, 0.8),(0.6, 0.3, 0.6)⟩ .

Then τ_N={ 0_N, A_1, A_2, A_3, A_4, 1_N } is Neutrosophic topological spaces on X.

Now, A_5 =⟨(0.5, 0.7, 0.5),(0.9, 0.4, 0.5)⟩ is Neutrosophic generalized semi closed set

Definition 3.3

Let(X, τ_N) be a Neutrosophic topological spaces . Then for a Neutrosophic subset A of X, the Neutrosophic semi-interior of A is the union of all Neutrosophic semi-open sets of X contained in A.

.i.e.Neu-GS-Int(A)

=∪{G :G is a Neu-GS open set in X and *G ⊆A}.*

Proposition 3.4

Neutrosophic subsets A and B of a Neutrosophic topological spaces X we have

(i) Neu-GS-Int(A)⊆A

(ii) A is Neu-GS-open set in X ⟺ Neu-GS-Int(A)=A

(iii) Neu-GS-Int(Neu-GS-Int(A))=Neu-GS-Int(A)

(iv) If A ⊆B then Neu-GS-Int(A)⊆Neu-GS-Int(B)

Proof:

(i) follows from Definition 3.3.

Let A be Neu-GS-open set in X. Then A⊆Neu-GS-Int(A). By using(i) we get A=Neu-GS-Int(A). Conversely assume that A=Neu-GS-Int(A). By using Definition 3.3, A is Neu-GS-open set in X. Thus (ii)is proved. By using (ii), Neu-GS-Int(Neu-GS-Int(A)) = Neu-GS-Int(A). This proves (iii). Since A ⊆ B, by using(i), Neu-GS-Int(A)⊆A⊆ B. That is Neu-GS-Int (A)⊆B. By(iii), Neu-GS-Int(Neu-GS-Int(A))⊆Neu-GS -Int(B).Thus Neu-GS-Int(A)⊆Neu-GS-Int(B). This proves (iv).

Theorem 3.5

Let(X, τ_N) be a Neutrosophic topological spaces . Then for any Neutrosophic subset A and B of a Neutrosophic topological spaces , we have

(i) Neu-GS-Int(A∩B)=Neu-GS-Int(A) ∩ Neu-GS-Int(B)

(ii) Neu-GS-Int(A∪B)⊇Neu-GS-Int(A)∪Neu-GS-Int(B).

Proof : Since A∩B ⊆ A and A∩B ⊆ B, by using Proposition 3.4(iv), Neu-GS-Int(A∩B) ⊆ Neu-GS-Int(A) and Neu-GS-Int(A∩B) ⊆ Neu-GS-Int(B). This implies that Neu-GS-Int(A∩B) ⊆ Neu-GS-Int(A) ∩Neu-GS-Int(B) -----(1).

By using Proposition 3.4(i), Neu-GS-Int(A) ⊆ A and Neu-GS-Int(B) ⊆ B. This implies that Neu-GS-Int(A) ∩ Neu-GS-Int(B) ⊆ A∩B . Now applying Proposition 3.4(iv), Neu-GS-Int((Neu-GS-Int(A) ∩ Neu-GS-Int (B)) ⊆ Neu-GS-Int(A∩B). By(1), Neu-GS-Int(Neu-GS-Int(A)) ∩ Neu-GS-Int(Neu-GS-Int(B)) ⊆ Neu-GS-Int(A∩B). By Proposition 3.2(iii), Neu-GS-Int(A) ∩ Neu-GS-Int(B) ⊆Neu-GS-Int(A∩B) -----(2).

From(1) and(2), Neu-GS-Int(A∩B) = Neu-GS-Int(A) ∩ Neu-GS-Int(B). This implies(i).Since A ⊆ A ∪ B and B ⊆A∪B, by using Proposition 3.4(iv), Neu-GS-Int(A)⊆Neu-GS-Int(A ∪ B) and Neu-GS-Int(B)⊆Neu-GS-Int(A ∪ B). This implies that Neu-GS-Int(A)∪Neu -GS-Int(B)⊆Neu-GS-Int(A∪B). Hence(ii).

The following example shows that the equality need not be hold in Theorem 3.5(ii).

Example 3.6

Let X ={ a,b,c} and τ_N={ 0_N, A_1, A_2, A_3, A_4, 1_N } where

A_1 = ⟨(0.41, 0.71, 0.11),(0.51, 0.61, 0.21), (0.91, 0.71, 0.31)⟩,

A_2 = ⟨(0.41, 0.61, 0.11),(0.71, 0.71, 0.21), (0.91, 0.51, 0.11)⟩,

A_3 = ⟨(0.41, 0.71, 0.11),(0.71, 0.71, 0.21), (0.91, 0.71, 0.11)⟩,

A_4 = ⟨(0.41, 0.61, 0.11),(0.51, 0.61, 0.21), (0.91, 0.51, 0.31)⟩.

Then(X, τ_N) is a Neutrosophic topological spaces.

Consider the Neutrosophic sets are

E = ⟨(0.71, 0.61, 0.11),(0.71, 0.61, 0.11), (0.91, 0.51, 0.01)⟩ and

F =⟨(0.41, 0.61, 0.11),(0.51, 0.71, 0.21), (2.1, 0.71, 0.11)⟩.

Then Neu-GS-Int(E) =D and Neu-GS-Int(F) = D. This implies that Neu-GS-Int(E)∪Neu-GS-Int(F)=D.

Now,

E ∪ F = ⟨(0.71, 0.61, 0.11),(0.71, 0.71, 0.11), (2.01, 0.71, 0.01)⟩,

it follows that Neu-GS-Int(E ∪ F) = B. Then Neu-GS-Int(E ∪ F) ⊈ Neu-GS-Int(E)∪Neu-GS-Int(F).

4.NEUTROSOPHIC GENERALIZED SEMI-CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the concept of Neutrosophic generalized semi closure operators in a Neutrosophic topological spaces.

Definition 4.1
Let(X, τ_N) be a Neutrosophic topological spaces. Then for a Neutrosophic subset A of X.
The Neutrosophic semi-closure of A is the intersection of all Neutrosophic generalized semi closed sets of X contained in A. That is,
Neu-GS-Cl(A)
=∩{ K :K is a Neu-GS-C set in X and K ⊇ A}.

Proposition 4.2
Let(X, τ_N) be a Neutrosophic topological spaces. Then for any Neutrosophic subsets A of X,
(i) C(Neu-GS-Int(A))=Neu-GS-Cl(C(A)),
(ii) C(Neu-GS-Cl(A))=Neu-GS-Int(C(A)).

Proof :
By using Definition 3.3,
Neu-GS-Int(A) = ∪{G :G is a Neu-GS-open set in X and G ⊆ A }. Taking complement on both sides, C(Neu-GS-Int(A)) = C(∪ {G :G is a Neu-GS open set in X and G⊆A})=∩{C(G):C(G) is a Neu-GS-C set in X and C(A) ⊆ C(G) }. Replacing C(G) by K, we get C(Neu-GS-Int(A))=∩{ K:K is a Neu-GS-C set in X and K ⊇ C(A)}. By Definition 4.1, C(Neu-GS-Int(A)) =Neu-GS-Cl(C(A)). This proves(i). By using(i), C(Neu-GS-Int(C(A)))=Neu-GS-Cl(C(C(A)))=Neu-GS-Cl(A). Taking complement on both sides, we get Neu-GS-Int(C(A))=C(Neu-GS-Cl(A)).
Hence proved(ii)

Proposition 4.3
Let(X, τ_N) be a Neutrosophic topological spaces . Then for any Neutrosophic subsets A and B of a Neutrosophic topological spaces X we have
(i) A ⊆ Neu-GS-Cl(A)
(ii) A is Neu-GS-C set in X ⇔ Neu-GS-Cl(A)=A
(iii) Neu-GS-Cl(Neu-GS-Cl(A))=Neu-GS-Cl(A)
(iv) If A ⊆ B then Neu-GS-Cl(A)⊆Neu-GS-Cl(B)

Proof :
(i) follows from Definition 4.2.
Let A be Neu-GS-closed set in X. By using Proposition 4.3,C(A) is Neu-GS-open set in X.By Proposition 4.2(ii), Neu-GS-nt(C(A))=C(A)⇔C(Neu-GS-Cl(A))=C(A)⇔Neu-GS-Cl(A)=A.Thus proved(ii) .By using(ii), Neu-GS-Cl(Neu-GS-Cl(A))=Neu-GS-Cl(A) .This proves(iii).Since A⊆B,C(B)⊆C(A). By using Proposition 3.4(iv),Neu-GS-Int(C(B))⊆Neu-GS-Int(C(A)). Taking complement on both sides, C(Neu-GS-Int(C(B)))⊇C(Neu-GS-Int(C(A))). By Proposition 4.2(ii),Neu-GS-Cl(A)⊆Neu-GS-Cl(B).This proves(iv).

Proposition 4.4
Let A be a Neutrosophic set in a Neutrosophic topological spaces X. Then Then Neu-Int(A)⊆Neu-GS-Int(A)⊆A⊆Neu-GS-Cl(A) ⊆Neu-Cl(A).

Proof :
It follows from the definitions of corresponding operators

Proposition 4.5
Let(X, τ_N) be a Neutrosophic topological spaces . Then for a Neutrosophic subset A and B of a Neutrosophic topological spaces X, we have
(i) Neu-GS-Cl(A∪B) =Neu-GS-Cl(A)∪Neu-GS-Cl(B) and
(ii) Neu-GS-Cl(A∩B)⊆Neu-GS-Cl(A)∩Neu-GS-Cl(B).

Proof :
Since Neu-GS-Cl(A∪B)=Neu-GS-Cl(C(C(A∪B))), By usingProposition4.2(i),Neu-GS-Cl(A∪B)=C(Neu-GS-Int(C(A∪B)))=C(Neu-GS-Int(C(A)∩C(B))). Again using Proposition3.5(i),Neu-GS-Cl(A∪B) =C(Neu-GS-Int(C(A))∩Neu-GS-Int(C(B)))=C(Neu-GS-Int(C(A)))∪C(Neu-GS-Int(C(B))).By using Proposition 4.2(i),Neu-GS-Cl(A∪B)=Neu-GS-Cl (C(C(A)))∪Neu-GS-Cl(C(C(B)))= Neu-GS-Cl (A)∪Neu-GS-Cl(B).Thus proved(i). Since A∩B ⊆A and A∩B ⊆ B, by using Proposition 4.3(iv), Neu-GS-Cl(A∩B)⊆Neu-GS-Cl(A) and Neu-GS-Cl (A∩B) ⊆ Neu-GS-Cl(B). This implies that Neu-GS-Cl(A∩B)⊆Neu-GS-Cl(A)∩Neu-GS-Cl(B).This proves(ii).
The following example shows that the equality need not be hold in Proposition 4.5(ii).

Example 4.6
Let X = { a , b ,c} with τ_N ={ 0_N, A_1, A_2, A_3, A_4, 1_N } and
C(τ_N)={1_N, A_5, A_6, A_7, A_8,0_N } where
A_1 = ⟨(0.5, 0.6, 0.1),(0.6, 0.7, 0.1),(0.9, 0.5, 0.2)⟩
A_2= ⟨(0.4, 0.5, 0.2),(0.8, 0.6, 0.3),(0.9, 0.7, 0.3)⟩
A_3 = ⟨(0.4, 0.5, 0.2),(0.6, 0.6, 0.3),(0.9, 0.5, 0.3)⟩
A_4 = ⟨(0.5, 0.6, 0.1),(0.8, 0.7, 0.1),(0.9, 0.7, 0.2)⟩
A_5 = ⟨(0.1, 0.4, 0.5),(0.1, 0.3, 0.6),(0.2, 0.5, 0.9)⟩,
A_6 = ⟨(0.2, 0.5, 0.4),(0.3, 0.4, 0.8),(0.3, 0.3, 0.9)⟩,
A_7 = ⟨(0.2, 0.5, 0.4),(0.3, 0.4, 0.6),(0.3, 0.5, 0.9)⟩,
A_8 = ⟨(0.1, 0.4, 0.5),(0.1, 0.3, 0.8),(0.2, 0.3, 0.9)⟩.
Then(X, τ_N) is a Neutrosophic topological spaces.
Consider the Neutrosophic sets are
A_9 = ⟨(0.1, 0.2, 0.5),(0.2, 0.3, 0.7),(0.3, 0.3, 1)⟩
and
A_{10} = ⟨(0.2, 0.4, 0.8),(0.1, 0.2, 0.8),(0.2, 0.5, 0.9)⟩.
Then Neu-GS-Cl(A_9)=A_7 and Neu-GS-Cl(A_{10})=A_7.
This implies that Neu-GS-Cl(A_9)∩Neu-GS-Cl(A_{10}) = A_7. Now, A_9 ∩ A_{10} = ⟨(0.1, 0.2, 0.8),(0.1,0.2,0.8), (0.2, 0.3,1)⟩, it follows that Neu-GS-Cl(A_9∩A_{10})= A_8. Then Neu-GS-Cl(A_9) ∩Neu-GS-Cl(A_{10}) ⊈ Neu-GS-(A_9∩A_{10}).

Theorem 4.7
Let(X, τ_N) be a NTS. Then for a Neutrosophic subset A and B of X we have
(i) Neu-GS-Cl(A)⊇ A∪ Neu-GS-Cl(Neu-GS-Int(A)),
(ii) Neu-GS-Int(A)⊆A∩Neu-GS-Int(Neu-GS-Cl(A)),
(iii) Neu-Int(Neu-GS-Cl(A)) ⊆ Neu-Int(Neu-Cl(A)),
(iv) Neu-Int(Neu-GS-Cl(A)) ⊇ Neu-Int(Neu-GS-Cl(Neu-GS-Int(A))).

Proof :

By Proposition 4.3(i), A ⊆ Neu-GS-Cl(A)---(1). Again using Proposition 3.4(i), Neu-GS-Int(A) ⊆ A.Then Neu-GS-Cl(Neu-GS-Int(A))⊆Neu-GS-Cl(A) ,(2).
By(1) &(2) we have, A ∪ Neu-GS-Cl(Neu-GS-Int (A))⊆Neu-GS-Cl(A). This proves(i).
By Proposition 3.4(i), Neu-GS-Int(A) ⊆ A -----(3). Again using proposition 4.3(i), A ⊆ Neu-GS-Cl(A). Then Neu-GS-Int(A)⊆Neu-GS-Int(Neu-GS-Cl(A))--(4).From(3) &(4), we have Neu-GS-Int(A) ⊆ A∩Neu -GS-Int(Neu-GS-Cl(A)). This proves(ii).
By Proposition 4.4, Neu-GS-Cl(A) ⊆ Neu-Cl(A).We get Neu-Int(Neu-GS-Cl(A)) ⊆ Neu-Int(Neu-Cl(A)). Hence(iii).By(i), Neu-GS-Cl(A) ⊇ A ∪ Neu-GS-Cl (Neu-GS-nt(A)). We have Neu-Int(Neu-GS-Cl(A) ⊇Neu-Int(A∪Neu-GS-Cl(Neu-GS-Int(A))).Since Neu-Int(A∪B)⊇Neu-Int(A)∪Neu-Int(B),Neu-Int(Neu-GS-Cl(A)⊇Neu-Int(A)∪Neu-Int(Neu-GS-l(Neu-GS-Int (A)))⊇Neu-Int(Neu-GS-Cl(Neu-GS-Int(A))).
Hence(iv).

Theorem 4.8

Let(X, τ_N) be a Neutrosophic topological spaces. Then for a Neutrosophic subset A and B of X we have,
(i) Neu-GS-Cl(A)⊇ A∪Neu-GS-Cl(Neu-GS-Int(A)),
(ii) Neu-GS-Int(A) ⊆ A ∩ Neu-GS-Int(Neu-GS-Cl(A)),
(iii) Neu-Int(Neu-GS-Cl(A))⊆Neu-Int(Neu-Cl(A)),
(iv)Neu-Int(Neu-GS-Cl(A))⊇Neu-Int(Neu-GS-Cl (Neu-GS-Int(A))).

Proof:

By Proposition 4.3(i), A ⊆ Neu-GS-Cl(A) -----(1). Again using Proposition 3.4(i), Neu-GS-Int(A) ⊆ A. ThenNeu-GS-Cl(Neu-GS-Int(A))⊆Neu-GS-Cl(A)--(2)
By(1)&(2) we have, A∪Neu-GS-Cl(Neu-GS-Int(A)) ⊆Neu-GS-Cl(A). This proves(i).By Proposition 3.4(i), Neu-GS-Int(A)⊆A -----(1). Again using proposition 4.3(i), A⊆Neu-GS-Cl(A). ThenNeu-GS-Int(A)⊆ Neu-GS-Int(Neu-GS-Cl(A)) -----(2).
From(1) &(2), we have Neu-GS-Int(A)⊆A∩Neu-GS-Int(Neu-GS-Cl(A)).This proves(ii).By Proposition 4.4, Neu-GS-Cl(A)⊆Neu-Cl(A).We get Neu-Int(Neu-GS-Cl(A))⊆Neu-Int(Neu-Cl(A)).Hence(iii).By(i),Neu-GS -Cl(A)⊇A∪Neu-GS-(Neu-GS-Int(A)).We have Neu-Int(Neu-GS-Cl(A)⊇Neu-Int(A∪Neu-GS-Cl(Neu-GS-Int(A))).SinceNeu-Int(A∪B)⊇Neu-Int(A)∪Neu-Int(B) ,Neu-int(Neu-GS-Cl(A)⊇Neu-Int(A)∪Neu-Int(Neu-GS-Cl(Neu-GS-Int(A)))⊇Neu-Int(Neu-GS-Cl(Neu-GS-Int(A))).Hence(iv).

REFERENCES

[1] K. Atanassov,Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986),87-94.
[2] C.L. Chang, Fuzzy Topological Spaces, J.Math. Anal. Appl.24(1968), 182-190.
[3] Dogan Coker, An introduction tointuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1997), 81-89.
[4] R.Dhavaseelan and S.Jafari,Generalized Neutros ophic closed sets, New trends in rosophic theory and applications VolumeII- 261-273,(2018)
[5]. P.Deepika, S.Chandrasekar "Fine GS Closed Sets and Fine SG Closed Sets in Fine topological Space", International Journal of Mathematics Trends and Technology (IJMTT). V56(1):8-14 April 2018
[6]. P.Deepika, S.Chandrasekar ,F-gs Open , F-gslosed and F-gs Continuous Mappings in Fine Topological Spaces, International Journal of Engineering,Science and Mathematics,Vol.7 (4),April 2018, 351-359
[7]. P.Deepika, S.Chandrasekar , M. Sathyabama, Properties Of Fine Generalized Semi Closed Sets In Fine Topological Spaces ,Journal Of Computer And Mathematical Sciences ,Vol 9 No.4, April (2018) ,293-301
[8] Florentin Smarandache , Neutrosophic and Neutrosophic Logic,FirstInternational Confer on Neutrosophic , Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002) ,smarand@unm.edu
[9] Floretin Smaradache, Neutrosophic Set :- A Generalization of Intuitionistic Fuzzy set, Journal of Defense Resourses Management.1(2010), 107-114.
[10] P.Iswarya and Dr.K.Bageerathi,On Neutrosophic semi-open sets in Neutrosophic topological spaces, International Journal of Mathematics Trends and Technology(IJMTT),Vol37,No.3 (2016), 24-33.
[11] A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Journal computer Sci. Engineering, Vol.(2) No.(7)(2012).
[12] A.A.Salama and S.A.Alblowi, Neutrosophic set and Neutrosophic topological space,SOR. mathematics, Vol.(3),Issue(4),(2012).pp-31-35
[13] L.A. Zadeh, Fuzzy Sets, Inform and Control 8(1965), 338- 353.